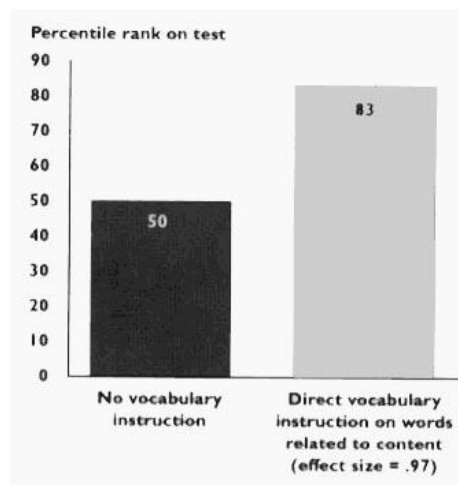


Vocabulary in Focus

This vocabulary section is designed to help systematically enhance the academic vocabulary of students to better prepare them to learn new content in mathematics. The research and theory underlying the recommendations made here have been detailed in the book *Building Background Knowledge for Academic Achievement* (Marzano, 2004). The logic of such an endeavor is that the more general background knowledge a student has about academic content addressed in a given class or course, the easier it is for the student to understand and learn.

The bar on the left side of the graph shows a student at the 50th percentile in terms of ability to comprehend the subject matter taught in school with no direct vocabulary instruction. The bar on the right side shows the comprehension level of the same student after vocabulary terms have been taught in a specific way. The dramatic increase to 83% provides a strong argument for teaching academic vocabulary. Because of a variety of factors, there is typically great disparity in the academic background knowledge of students. This disparity increases as students progress through the school years. However, if all students were exposed to specific academic terms across the grade levels, a strong common foundation



for all students would be formed. To this end, this section lists important academic terms in mathematics. The words listed in this document are not all inclusive, but are suggested as a starting point in building the academic vocabulary for a given grade or course.

The following table provides an overview of the number of terms in each grade:

	Number of words		Number of words
Grade K	43	Grade 6	34
Grade 1	40	Grade 7	31
Grade 2	34	Grade 8	28
Grade 3	41	Algebra I / Technical Algebra	26
Grade 4	40	Algebra II	43
Grade 5	28	Geometry / Technical Geometry	28

To demonstrate the potential power of addressing common terms and phrases, there are 326 terms listed for grades K – 8. If every teacher were to teach these terms, students would enter ninth grade with common, in depth experiences with these key mathematics terms. Certainly this would provide a strong base on which ninth grade mathematics teachers could build.

A five-step process

There is no single best way to teach terms and phrases. However, research and theory on vocabulary development point to a few generalizations that provide strong guidance.

1. Initially Provide Students with a Description, Explanation, or Example as Opposed to a Formal Definition

When introducing a new term or phrase it is useful to avoid a formal definition at the start. Formal definitions are typically not very “learner friendly.” They make sense after there is a general understanding of a term. Provide students with a description, explanation, or example. Ask students what they already know to avoid misconceptions.

2. Have Students Generate Their Own Descriptions, Explanations, or Examples

Once a description, explanation, or example has been provided to students, they should be asked to restate that information in their own words. It is important that students do not copy exactly what the teacher has offered. Student descriptions, explanations, and examples should be their own constructions using their own background knowledge and experiences to forge linkages between the new term or phrase and what they already know.

3. Have Students Represent Each Term or Phrase Using a Graphic Representation, Picture, or Pictograph

Once students have generated their own description, explanation, or example they should be asked to represent the term or phrase in some graphic, picture, or pictographic form. This allows them a different, nonlinguistic way to process the information. It also provides a second processing of the information which should help deepen students’ understanding of the new term or phrase.

4. Have Students Keep an Academic Vocabulary Notebook

Over time students will develop an understanding of a set of terms and phrases that are important to the academic content in mathematics. This implies that the terms and phrases that are taught using this approach represent a related set of knowledge that expands and deepens from year to year.

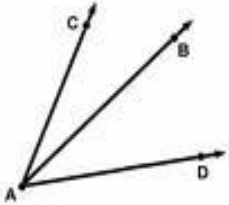
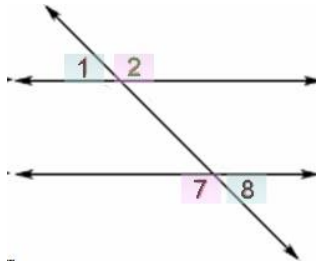
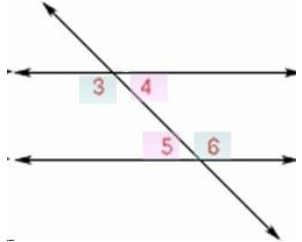
To facilitate this cumulative effect it is highly advisable for students to keep an “academic vocabulary” notebook that contains the terms and phrases that have been taught. Enough space should be provided for students to record their initial descriptions, explanations, and examples of the terms and phrases as well as their graphic representations, pictures, and pictographs.

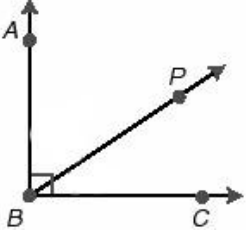
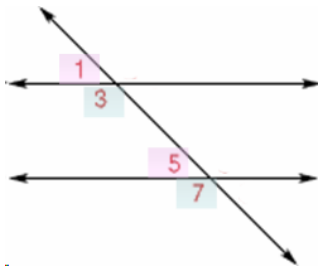
Students should be engaged in activities that allow them to review the terms in their academic vocabulary notebooks and add to their knowledge base.

5. Periodically Review the Terms and Phrases and Provide Students with Activities That Add to Their Knowledge Base

If students experience a new term or phrase once only, they will be left with their initial, partial understanding of the term or phrase. To develop deep understanding of the terms must be engaged in review activities. Offer students activities that add to their knowledge base about the terms in their notebooks. For example, they might make comparisons between selected terms; they might create analogies or metaphors for selected terms; they might simply compare their entries with those of other students. Finally, they might be engaged in games that use the terms from their academic vocabulary notebooks. After each of these activities students should be asked to make corrections, additions, and changes to the entries in their notebooks. In this way, students’ knowledge of the academic terms and phrases deepen and become a sound foundation on which to build the academic content presented in class.

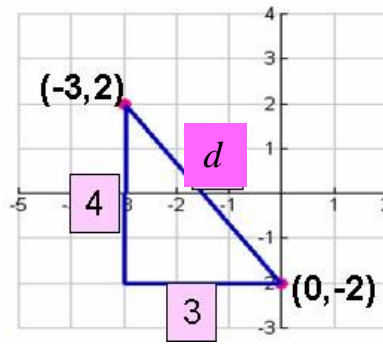
Eighth Grade

<p>Accuracy: The closeness of a given measurement or value to the true measurement or value. How closely a measured value agrees with the correct value. (Precision refers to how closely individual measurements agree with each other. So, the smaller the unit of measurement, the better the precision.)</p> <p>If you are measuring a piece of string and do not stretch it out straight, you could have good precision, but poor accuracy.</p>	<p>Example: Two students measure the length of a board. One student uses a ruler with $\frac{1}{4}$ inch intervals (precision: $\frac{1}{4}$ in). The other student uses a ruler with one centimeter units (precision: 1cm). The first student measured the board to be 100 inches, while the second measured 254 centimeters. Which student has the more accurate measurement?</p> <p>Since the greatest possible error is one-half the precision of the instrument, the first student has a relative error of $\frac{\frac{1}{8}''}{100''} = .125\%$. The second student's greatest possible error is .5 cm, so the relative error is $\frac{.5cm}{254cm} = .2\%$</p> <p>The first student has higher accuracy.</p>
<p>Adjacent Angles: Angles that have a common vertex and a common side.</p>	<p>$\angle CAB$ and $\angle BAD$ are adjacent. Ray AB is their common side and point A is their common vertex.</p> 
<p>Alternate Exterior Angle: A pair of angles on the outer sides of two lines cut by a transversal, but on opposite sides of the transversal</p>	<p>If the lines are parallel:</p> <p>$\angle 1$ and $\angle 8$ are alternate exterior angles and are congruent.</p> <p>$\angle 2$ and $\angle 7$ are also alternate exterior angles and are congruent.</p> 
<p>Alternate Interior Angle: A pair of angles on the inner sides of two lines cut by a transversal, but on the opposite sides of the transversal.</p>	<p>If the lines are parallel</p> <p>$\angle 3$ and $\angle 6$ are alternate interior angles and are congruent.</p> <p>$\angle 4$ and $\angle 5$ are also alternate interior angles and are congruent</p> 

<p>Complementary Angle: Two angles that add up to 90 degrees. (Their sum is 90.)</p>	 <p>Angles ABP and PBC are complementary.</p> $\angle ABP + \angle PBC = 90^\circ$
<p>Compound Event: An event whose probability depends on the occurrence of two or more events such as two socks being drawn from a drawer.</p> <p>To find the probability of two <i>independent</i> events both occurring, multiply the probability of the first event by the probability of the second event: $P(A \text{ and } B) = P(A) \cdot P(B)$</p> <p>To find the probability of two <i>dependent</i> events both occurring, multiply the probability of A and the probability of B after A occurs: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$</p> <p>To find the probability of one or the other of two <i>mutually exclusive</i> events, add the probability of the first event to the probability of the second event: $P(A \text{ or } B) = P(A) + P(B)$</p>	<p>Dependent events (<i>and</i>): A drawer contains 4 blue, 6 black, and 2 brown socks. What is the probability that you choose two blue socks in a row?</p> $p(\text{blue and blue}) = \frac{4}{12} \cdot \frac{3}{11} = \frac{12}{132} = \frac{1}{11}$ <p>Independent events (<i>and</i>): If you roll two six-sided number cubes, what is the probability that you roll two even numbers?</p> $p(\text{even and even}) = \frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36} = \frac{1}{4}$ <p>Mutually exclusive events (<i>or</i>): If you roll a six-sided number cube, what is the probability that you roll a 1 or a 6?</p> $p(1 \text{ or } 6) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
<p>Corresponding Angle: Angles that are on the same side of the transversal and are both above or both below the lines cut by the transversal</p>	<p>If the lines are parallel:</p>  <p>$\angle 1$ and $\angle 5$ are corresponding and congruent.</p> <p>$\angle 3$ and $\angle 7$ are corresponding and congruent.</p>
<p>Cost Per Unit: A unit rate used to compare costs per single item; a rate in which the second quantity is one</p>	$\frac{\$3.90}{10 \text{ markers}} = \frac{\$0.39}{1 \text{ marker}}$ <p>Unit Cost = \$0.39</p>

Distance Formula (d = rt):

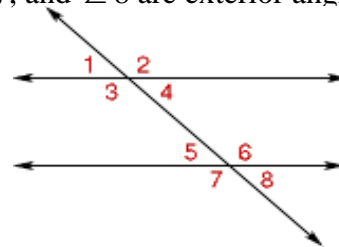
From the Pythagorean Theorem, the distance d between any two points (x_1, y_1) and (x_2, y_2) is $d^2 = a^2 + b^2$ or $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



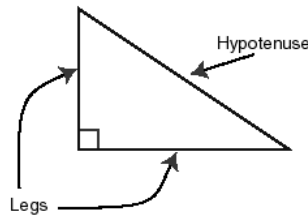
The distance $d = \sqrt{9+16} = \sqrt{25} = 5$

Exterior Angles: The angles on the outer sides of two lines cut by a transversal

$\angle 1, \angle 2, \angle 7,$ and $\angle 8$ are exterior angles.

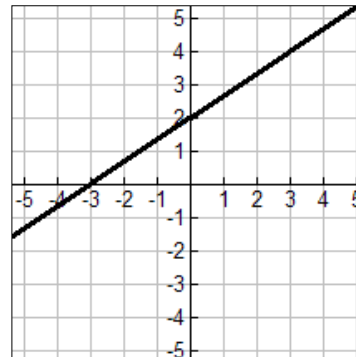


Hypotenuse: The longest side of a right triangle, or the side directly across from the right angle



Intercept: The point where a graph crosses either the x- or the y-axis. The y-intercept of the line $y = mx + b$ is b

The y-intercept can be found algebraically by letting $x = 0$ and solving for y. The x-intercept can be found algebraically by letting $y = 0$ and solving for x.

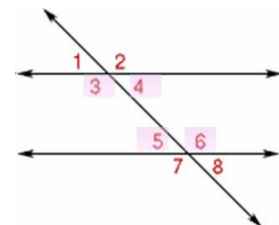


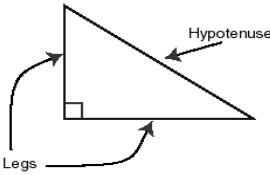
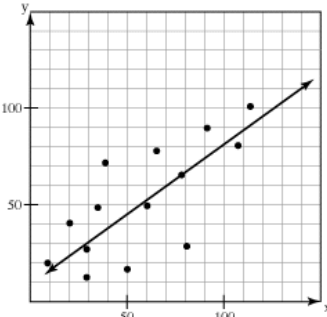
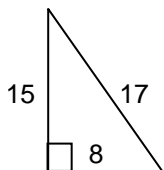
$$y = \frac{2}{3}x + 2$$

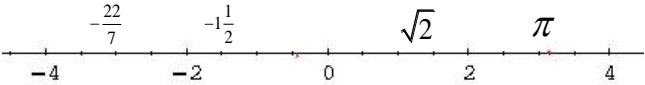
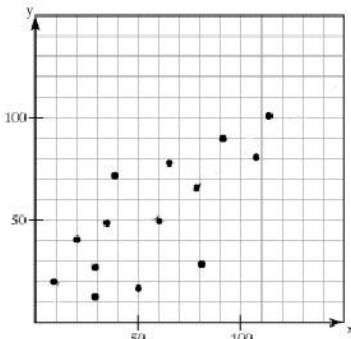
The y-intercept is 2.
The x-intercept is -3.

Interior Angles: Angles on the inner sides of two lines cut by a transversal

Angles 3, 4, 5 and 6 are interior angles.



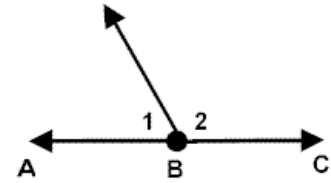
<p>Irrational Numbers: A number that cannot be written as a ratio of two integers. Irrational numbers in decimal form are non-terminating and non-repeating.</p>	<p>These numbers are irrational:</p> $\sqrt{2} = 1.414213562\dots$ $.01011011101111\dots$ $\pi \approx 3.14159265358979323\dots$
<p>Laws of Exponents: If you multiply two expressions with the same base, the base stays the same and you add the exponents: $b^x b^y = b^{x+y}$ If you divide two expressions with the same base, the base stays the same and you subtract the exponents: $\frac{b^x}{b^y} = b^{x-y}$ If you raise an expression with an exponent to a power, the base stays the same and you multiply the exponents: $(b^x)^y = b^{xy}$</p>	<p>Examples:</p> $(2^3)(2^2) = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^5 = 32$ $\frac{3^5}{3^2} = \frac{\cancel{3} \cdot \cancel{3} \cdot 3 \cdot 3 \cdot 3}{\cancel{3} \cdot \cancel{3}} = 3^3 = 27$ $(4^2)^3 = (4^2) \cdot (4^2) \cdot (4^2) = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^6 = 4096$ <p>If $x \neq 0$, $x^0 = 1$ (0^0 is undefined)</p> $\frac{3^2}{3^2} = 3^0 = 1$
<p>Legs of a Triangle: In a right triangle, the two sides that are not the hypotenuse (the longest side) or the two sides that form the right angle.</p>	 <p>In a right triangle: The sum of the squares of the legs is equal to the square of the hypotenuse.</p>
<p>Line of Best Fit (conceptual): A straight line that best fits the data on a scatter plot. It can be used to predict a trend in the data.</p>	
<p>Multi-step Equations: Equations that contain more than one operation and thus take more than one step (using inverse operations) to solve.</p>	<div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid green; padding: 5px;"> $3(x+1) = 15$ $3x + 3 = 15$ $3x = 12$ $x = 4$ </div> <div style="border: 1px solid red; padding: 5px;"> $2x - 3 = 17$ $2x = 20$ $x = 10$ </div> </div>
<p>Pythagorean Theorem: In a right triangle, the sum of the squares of the length of the legs is equal to the square of the length of the hypotenuse $a^2 + b^2 = c^2$</p>	 $8^2 + 15^2 = 17^2$ $64 + 225 = 289$

<p>Precision: The level of detail of a measurement, determined by the unit of measure. Precision depends on the smallest unit of measurement being used.</p>	<p>A meter stick that has centimeter markings has a precision of 1 cm and a possible error of plus or minus .5 cm. Significant digits can indicate the precision of a measurement. A measurement of 45.32 cm contains 4 significant digits. (The 2 is the estimated digit).</p> <p>A ruler with $\frac{1}{16}$" markings would have greater precision than a ruler with only $\frac{1}{4}$" markings.</p>
<p>Real Number: A number that is either rational or irrational. Real numbers can be represented by the set of infinite points on a number line.</p>	<p>Real numbers include all of the following numbers: rational, irrational, and thus integers, whole numbers, natural numbers, zero.</p> <p>The properties of real numbers include the commutative, associative, distributive, additive and multiplicative identity, and additive and multiplicative inverse properties.</p> 
<p>Relative Frequency: The observed number of successful events for a given number of trials: the ratio of the total number of times a given event occurs to the total number of events.</p> <p>The observed relative frequency is an approximation to the true probability of an event.</p>	<p>If we were able to perform a trial more and more times, the relative frequency would eventually approach the actual probability.</p> <p>For example: If you were to flip a coin 20 times, heads might come up 12 times. The relative frequency would be $\frac{12}{20} = .6$.</p> <p>Toss the coin 100 times, if there are 54 heads, the relative frequency would be $\frac{54}{100} = .54$.</p> <p>As you continued to increase the number of coin tosses, the relative frequency should approach the theoretical probability of .5.</p>
<p>Scatter Plot: A graph on a coordinate system used to display a set of data points</p>	

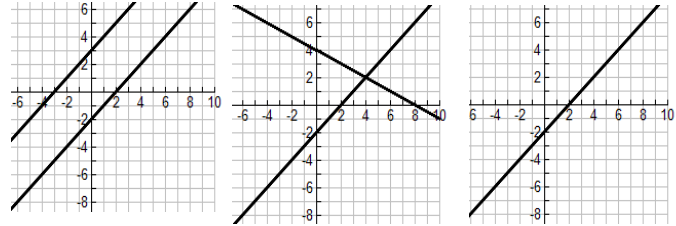
Supplementary Angle: Angles whose sums add up to 180 degrees

Angles 1 and 2 are supplementary.

$$\angle 1 + \angle 2 = 180^\circ$$



System of Equations: Two equations in two variables. The solution can be infinite (if they represent the same line), one point (x,y) (if the two lines intersect) or no solution (if the lines are parallel).



$$x - y = -3$$

$$x - y = 2$$

No solution:
Parallel lines

$$x + 2y = 8$$

$$x - y = 2$$

One solution:
(4,2)

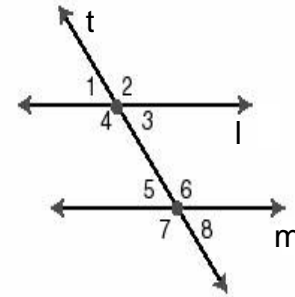
$$x - y = 2$$

$$2x - 2y = 4$$

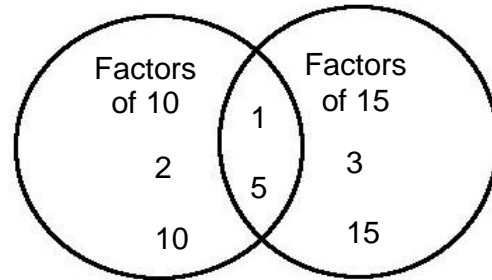
Infinite # of solutions:
same line

Transversal: A line that intersects two other lines (usually parallel).

Line t is a transversal which intersects parallel lines l and m .



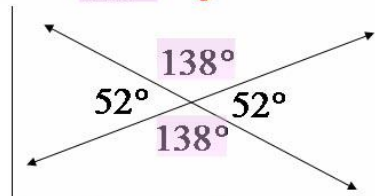
Venn Diagram: A diagram showing the relationships among sets of objects using overlapping circles.



This Venn diagram shows that the intersection of the factors of 10 and the factors of 15 are those they have in common, 1 and 5.

Vertical Angles: A pair of opposite congruent angles formed by intersecting lines

The two 138° angles are vertical



The two 52° angles are vertical