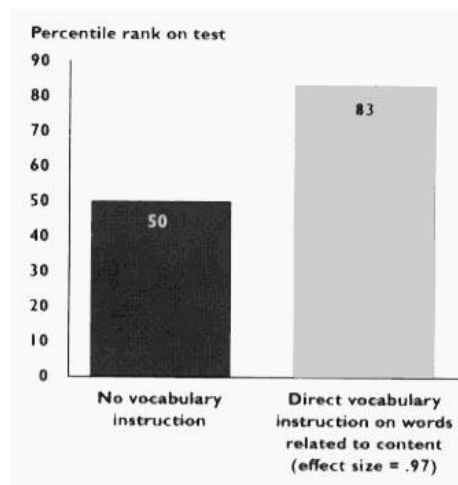


Vocabulary in Focus

This vocabulary section is designed to help systematically enhance the academic vocabulary of students to better prepare them to learn new content in mathematics. The research and theory underlying the recommendations made here have been detailed in the book *Building Background Knowledge for Academic Achievement* (Marzano, 2004). The logic of such an endeavor is that the more general background knowledge a student has about academic content addressed in a given class or course, the easier it is for the student to understand and learn.

The bar on the left side of the graph shows a student at the 50th percentile in terms of ability to comprehend the subject matter taught in school with no direct vocabulary instruction. The bar on the right side shows the comprehension level of the same student after vocabulary terms have been taught in a specific way. The dramatic increase to 83% provides a strong argument for teaching academic vocabulary. Because of a variety of factors, there is typically great disparity in the academic background knowledge of students. This disparity increases as students progress through the school years. However, if all students were exposed to specific academic terms across the grade levels, a strong common foundation



for all students would be formed. To this end, this section lists important academic terms in mathematics. The words listed in this document are not all inclusive, but are suggested as a starting point in building the academic vocabulary for a given grade or course.

The following table provides an overview of the number of terms in each grade:

	Number of words		Number of words
Grade K	43	Grade 6	34
Grade 1	40	Grade 7	31
Grade 2	34	Grade 8	28
Grade 3	41	Algebra I / Technical Algebra	26
Grade 4	40	Algebra II	43
Grade 5	28	Geometry / Technical Geometry	28

To demonstrate the potential power of addressing common terms and phrases, there are 326 terms listed for grades K – 8. If every teacher were to teach these terms, students would enter ninth grade with common, in depth experiences with these key mathematics terms. Certainly this would provide a strong base on which ninth grade mathematics teachers could build.

A five-step process

There is no single best way to teach terms and phrases. However, research and theory on vocabulary development point to a few generalizations that provide strong guidance.

1. Initially Provide Students with a Description, Explanation, or Example as Opposed to a Formal Definition

When introducing a new term or phrase it is useful to avoid a formal definition at the start. Formal definitions are typically not very “learner friendly.” They make sense after there is a general understanding of a term. Provide students with a description, explanation, or example. Ask students what they already know to avoid misconceptions.

2. Have Students Generate Their Own Descriptions, Explanations, or Examples

Once a description, explanation, or example has been provided to students, they should be asked to restate that information in their own words. It is important that students do not copy exactly what the teacher has offered. Student descriptions, explanations, and examples should be their own constructions using their own background knowledge and experiences to forge linkages between the new term or phrase and what they already know.

3. Have Students Represent Each Term or Phrase Using a Graphic Representation, Picture, or Pictograph

Once students have generated their own description, explanation, or example they should be asked to represent the term or phrase in some graphic, picture, or pictographic form. This allows them a different, nonlinguistic way to process the information. It also provides a second processing of the information which should help deepen students’ understanding of the new term or phrase.

4. Have Students Keep an Academic Vocabulary Notebook

Over time students will develop an understanding of a set of terms and phrases that are important to the academic content in mathematics. This implies that the terms and phrases that are taught using this approach represent a related set of knowledge that expands and deepens from year to year.

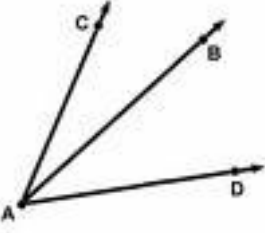
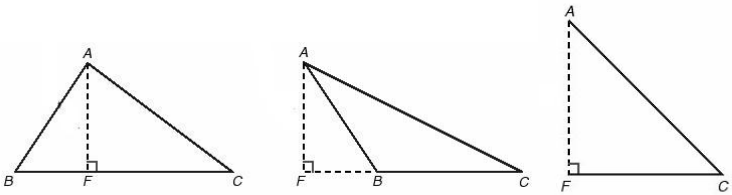
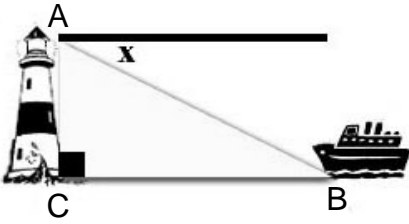
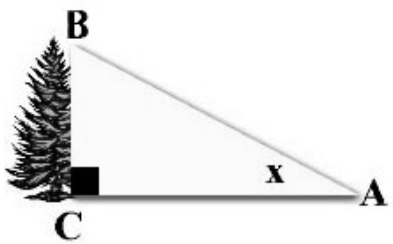
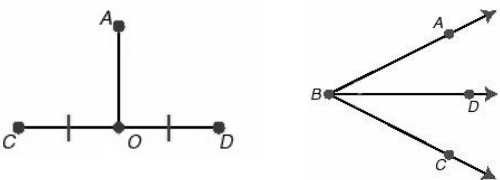
To facilitate this cumulative effect it is highly advisable for students to keep an “academic vocabulary” notebook that contains the terms and phrases that have been taught. Enough space should be provided for students to record their initial descriptions, explanations, and examples of the terms and phrases as well as their graphic representations, pictures, and pictographs.

Students should be engaged in activities that allow them to review the terms in their academic vocabulary notebooks and add to their knowledge base.

5. Periodically Review the Terms and Phrases and Provide Students with Activities That Add to Their Knowledge Base

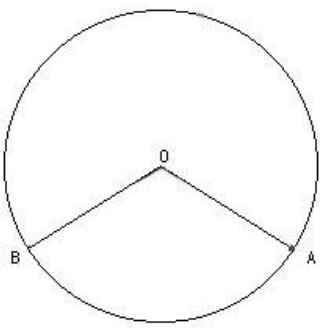
If students experience a new term or phrase once only, they will be left with their initial, partial understanding of the term or phrase. To develop deep understanding of the terms must be engaged in review activities. Offer students activities that add to their knowledge base about the terms in their notebooks. For example, they might make comparisons between selected terms; they might create analogies or metaphors for selected terms; they might simply compare their entries with those of other students. Finally, they might be engaged in games that use the terms from their academic vocabulary notebooks. After each of these activities students should be asked to make corrections, additions, and changes to the entries in their notebooks. In this way, students’ knowledge of the academic terms and phrases deepen and become a sound foundation on which to build the academic content presented in class.

Geometry / Technical Geometry

<p>Adjacent angles: Two angles in the same plane that are next to each other and share a common side and a common vertex.</p>	<p>$\angle CAB$ and $\angle BAD$ are adjacent. Ray AB is their common side and point A is their common vertex.</p> 
<p>Altitude of a triangle: The perpendicular segment from a vertex to the line containing the opposite side of a triangle.</p>	<p>In each triangle the altitude to side BC is segment AF:</p> 
<p>Angle of Depression: An angle formed by a horizontal line and the line of sight below it.</p>	 <p>Angle x is the angle of depression. It is congruent to $\angle ABC$</p>
<p>Angle of Elevation: An angle formed by a horizontal line and the line of sight above it.</p>	<p>Angle x is the angle of elevation.</p> 
<p>Bisect: To divide into two equal parts.</p> <p>An angle bisector is a ray in the interior of an angle that divides the angle into two congruent angles.</p> <p>A bisector of a segment contains the midpoint of the segment and divides it into two congruent segments. The bisector can be a line, a segment, a ray or a plane.</p>	 <p>Segment AO bisects Segment CD</p> <p>Ray BD bisects $\angle ABC$</p>

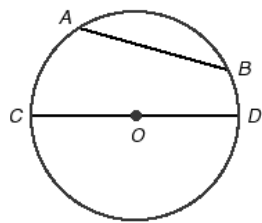
Central Angle of a circle: An angle in a circle whose vertex is the center of the circle and whose sides intersect the circle.

Angle BOA is a central angle.

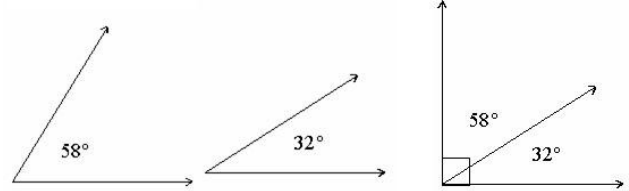


Chord of a circle: A segment whose endpoints are points on the circle. The longest chord of a circle is the diameter.

Segments AB and CD are chords of the circle.
The diameter \overline{CD} is the longest chord of the circle.

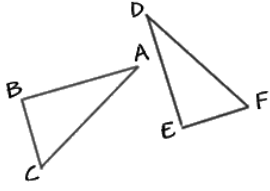


Complementary Angles: Two angles whose sum is 90 degrees.
 $\angle A$ and $\angle B$ are complementary if and only if $\angle A + \angle B = 90^\circ$.



A 58 degree angle and a 32 degree angle are complementary since $58 + 32 = 90$.
If complementary angles are adjacent, their exterior sides form a right angle.

Congruent figures: Two figures are congruent if they have exactly the same size and shape.
Two polygons are congruent if their corresponding sides and angles are congruent. The symbol for congruence is \cong .



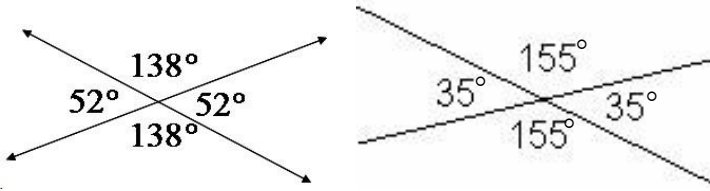
$$\triangle ABC \cong \triangle DEF$$

$$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{CA} \cong \overline{FD}$$

$$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$$

Conjecture: To make an educated guess or a prediction about future outcomes based on patterns, logic or data

After measuring several pairs of vertical angles, a conjecture was made that vertical angles are congruent. This conjecture used inductive reasoning.



Corresponding Parts of congruent or similar figures: A side (or an angle) of a polygon that is matched with a side (or an angle) of a congruent or a similar polygon.

If the polygons are congruent, the corresponding parts are congruent.

If the polygons are similar, the corresponding angles are congruent and the corresponding sides are in proportion.

$\triangle ABC \cong \triangle DEF$

The corresponding angles of the two similar triangles are congruent: $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

The corresponding sides are in proportion: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

Deductive Reasoning: Using facts, definitions, properties, axioms and theorems to reach a logical conclusion or to show that a conjecture is true. It is reasoning from the general to the specific.

- An example of deductive reasoning:
1. Given: Vertical angles are congruent
 2. $\angle A$ and $\angle B$ are vertical angles
 3. Therefore, $\angle A$ and $\angle B$ are congruent

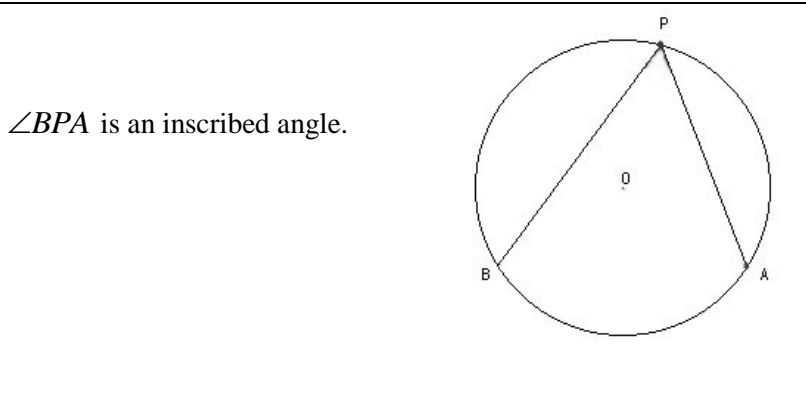
Geometric Mean: The geometric mean occurs in a proportion when the two inner terms are the same. If $\frac{a}{x} = \frac{x}{b}$ then $x^2 = ab$ and $x = \sqrt{ab}$; x is the geometric mean.

a - the first term (extreme)	$\frac{a}{x} = \frac{x}{b}$	x - the third term (mean)
x - the second term (mean)		b - the fourth term (extreme)

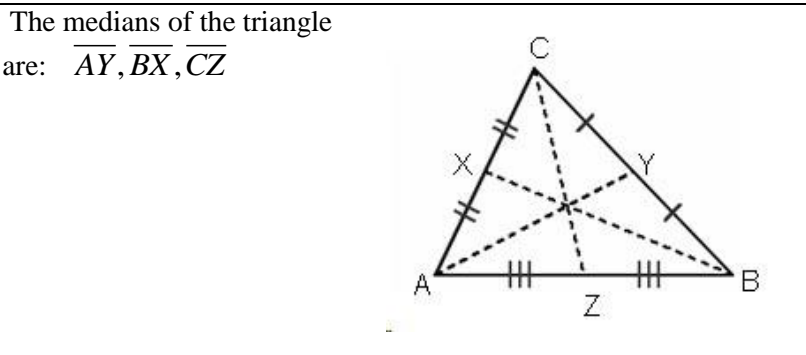
Inductive Reasoning: A conclusion or a prediction is reached based on patterns or many observations. It is reasoning from the specific to the general.

- An example of inductive reasoning:
1. A student measures several pairs of vertical angles
 2. Each time, the angles in the pair are congruent
 3. The student concludes that vertical angles are congruent.

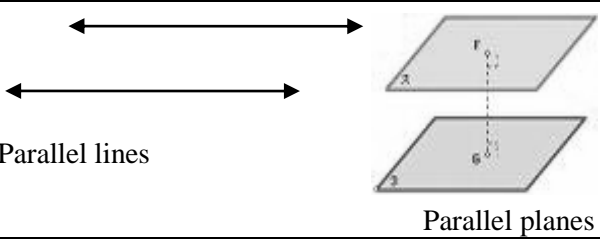
Inscribed Angle in a circle: An angle whose vertex lies on the circle and whose sides lie on chords of the circle.



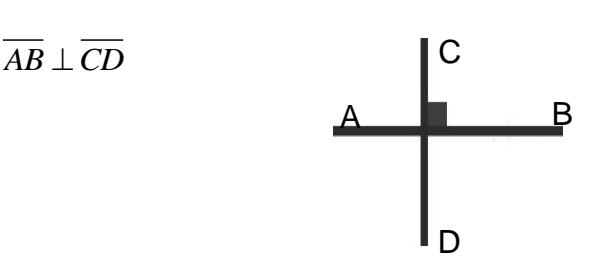
Median of a Triangle: A segment which connects the vertex of a triangle to the midpoint of its opposite side.



Parallel Lines: Lines that lie in a plane and do not intersect. (Parallel planes are planes which do not intersect)



Perpendicular: Lines, segments, rays or planes that intersect to form right angles.



Pi: The ratio of the circumference of a circle to the length of its diameter.

Approximations for pi are 3.14 and $\frac{22}{7}$



$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

$$\pi \approx 3.141592$$

$$535897932384$$

$$626433832795$$

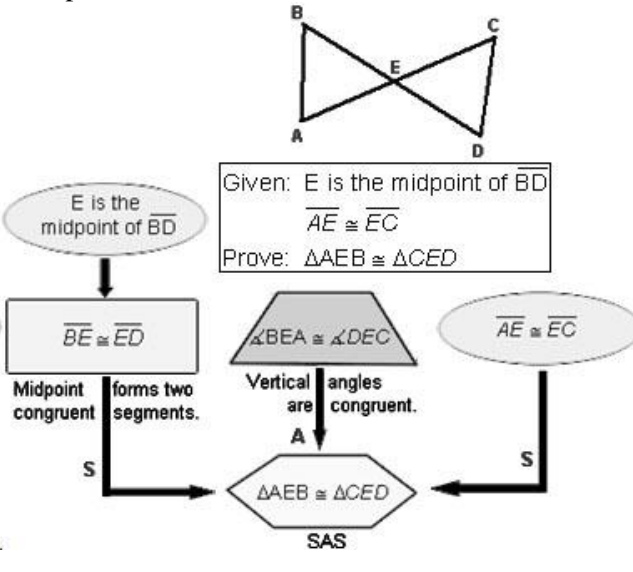
$$028841971693$$

$$9937510\ 58209$$

$$749445923078\dots$$

Proof:
Formal: A logical argument using statements supported by reasons, containing axioms, postulates, definitions and theorems in a chain of deductive reasoning.
Paragraph: A convincing argument that is written in complete sentences which starts with the hypothesis and ends with the conclusion.
Flow: A way to organize ideas in a proof using arrows to display the relationships between the statements in the proof.
Coordinate: A proof using ordered pairs and usually the distance formula, mid-point formula and/or the definition of slope to prove geometric conjectures.

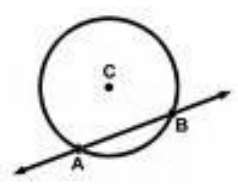
Example of Flow Proof:



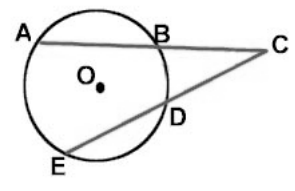
Properties :
Reflexive: Any segment or angle is congruent to itself.
Symmetric: If $a = b$, then $b = a$.
Transitive: If $a = b$ and $b = c$ then $a = c$.

Reflexive: $\angle A \cong \angle A$ or $AB \cong AB$
 Symmetric: $\angle A \cong \angle B$, therefore $\angle B \cong \angle A$
 Transitive: $\angle A \cong \angle B$, $\angle B \cong \angle C$, therefore $\angle A \cong \angle C$

Secant Line: A line that intersects a circle in exactly two points. A secant will contain a chord of the circle.

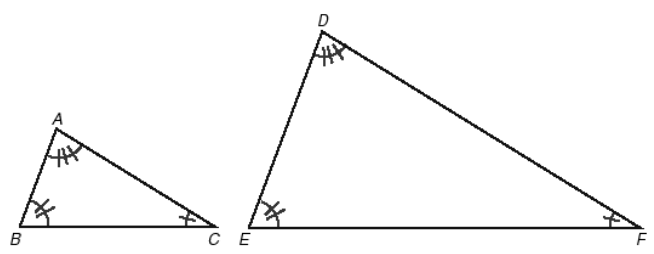


Secant \overline{AB}



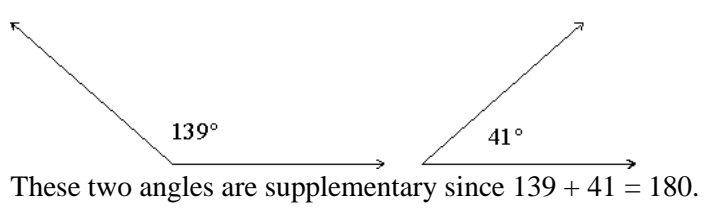
Angle ACE formed by secants CA and CE

Similar figures: Two figures are similar if their corresponding angles are congruent and their corresponding sides are in proportion. Similar figures have the same shape but different sizes. The symbol for similar is \square .

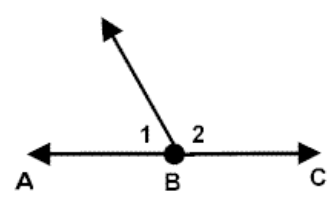


$\triangle ABC \square \triangle DEF \quad \angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$
 And $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Supplementary angles: Two angles whose sum is 180 degrees. $\angle A$ and $\angle B$ are supplementary if and only if $\angle A + \angle B = 180^\circ$

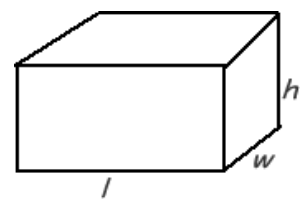


Angles 1 and 2 are supplementary.
 $\angle 1 + \angle 2 = 180^\circ$
 If supplementary angles are adjacent, their exterior sides form a straight line.



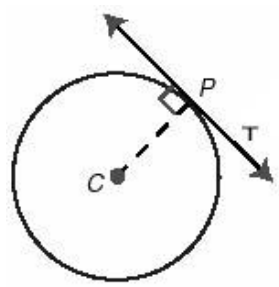
Surface Area of Solids: The surface area of a solid is the sum of the areas of the surfaces of the solid. The lateral area is the area of its lateral faces for a prism or pyramid and its curved lateral surface for a cylinder and a cone.

The surface area of the rectangular solid is
 $SA = 2lw + 2lh + 2wh$



Tangent Line: A line which intersects a figure or a solid in only one point.

\overline{PT} is tangent to circle C at point P. It is perpendicular to radius \overline{PC} .



Theorem: A statement which can be proved to be true.

Example: The Pythagorean Theorem, which states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse, can be proved many different ways.

Transversal: In a plane, a line that intersect two or more (usually parallel) lines.

Line t is a transversal which intersects parallel lines l and m .

