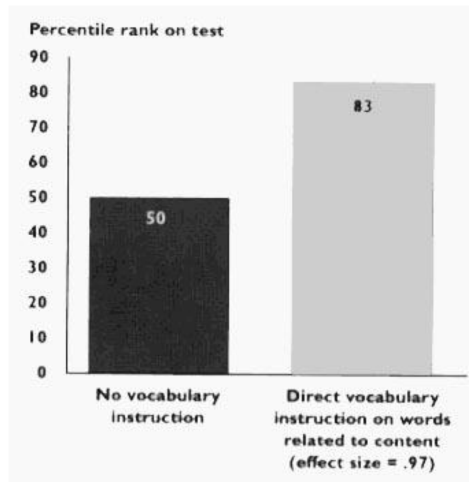
Vocabulary in Focus

This vocabulary section is designed to help systematically enhance the academic vocabulary of students to better prepare them to learn new content in mathematics. The research and theory underlying the recommendations made here have been detailed in the book *Building Background Knowledge for Academic Achievement* (Marzano, 2004). The logic of such an endeavor is that the more general background knowledge a student has about academic content addressed in a given class or course, the easier it is for the student to understand and learn.



The bar on the left side of the graph shows a student at the 50th percentile in terms of ability to comprehend the subject matter taught in school with no direct vocabulary instruction. The bar on the right side shows the comprehension level of the same student after vocabulary terms have been taught in a specific way. The dramatic increase to 83% provides a strong argument for teaching academic vocabulary. Because of a variety of factors, there is typically great disparity in the academic background knowledge of students. This disparity increases as students progress through the school years. However, if all students were exposed to specific academic terms across the grade levels, a strong common foundation for all students would be formed. To this end, this section lists important academic terms in mathematics. The words listed in this document are not all inclusive, but are suggested as a starting point in building the academic vocabulary for a given grade or course.

The following table provides an overview of the number of terms in each grade:

	Number of words			**Number of words**
Grade K	43		**Grade 6**	34
Grade 1	40		**Grade 7**	31
Grade 2	34		**Grade 8**	28
Grade 3	41		**Algebra I / Technical Algebra**	26
Grade 4	40		**Algebra II**	43
Grade 5	28		**Geometry / Technical Geometry**	28

To demonstrate the potential power of addressing common terms and phrases, there are 326 terms listed for grades K – 8. If every teacher were to teach these terms, students would enter ninth grade with common, in depth experiences with these key mathematics terms. Certainly this would provide a strong base on which ninth grade mathematics teachers could build.

A five-step process

There is no single best way to teach terms and phrases. However, research and theory on vocabulary development point to a few generalizations that provide strong guidance.

1. **Initially Provide Students with a Description, Explanation, or Example as Opposed to a Formal Definition**

When introducing a new term or phrase it is useful to avoid a formal definition at the start. Formal definitions are typically not very "learner friendly." They make sense after there is a general understanding of a term. Provide students with a description, explanation, or example. Ask students what they already know to avoid misconceptions.

2. **Have Students Generate Their Own Descriptions, Explanations, or Examples**

Once a description, explanation, or example has been provided to students, they should be asked to restate that information in their own words. It is important that students do not copy exactly what the teacher has offered. Student descriptions, explanations, and examples should be their own constructions using their own background knowledge and experiences to forge linkages between the new term or phrase and what they already know.

3. **Have Students Represent Each Term or Phrase Using a Graphic Representation, Picture, or Pictograph**

Once students have generated their own description, explanation, or example they should be asked to represent the term or phrase in some graphic, picture, or pictographic form. This allows them a different, nonlinguistic way to process the information. It also provides a second processing of the information which should help deepen students' understanding of the new term or phrase.

4. **Have Students Keep an Academic Vocabulary Notebook**

Over time students will develop an understanding of a set of terms and phrases that are important to the academic content in mathematics. This implies that the terms and phrases that are taught using this approach represent a related set of knowledge that expands and deepens from year to year.

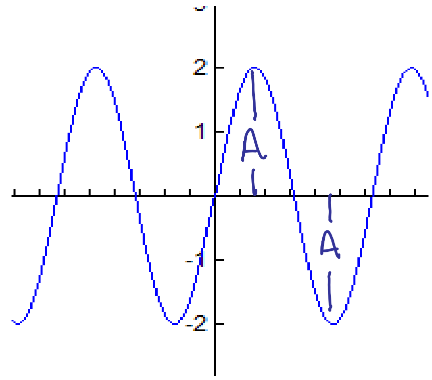
To facilitate this cumulative effect it is highly advisable for students to keep an "academic vocabulary" notebook that contains the terms and phrases that have been taught. Enough space should be provided for students to record their initial descriptions, explanations, and examples of the terms and phrases as well as their graphic representations, pictures, and pictographs.

Students should be engaged in activities that allow them to review the terms in their academic vocabulary notebooks and add to their knowledge base.

5. **Periodically Review the Terms and Phrases and Provide Students with Activities That Add to Their Knowledge Base**

If students experience a new term or phrase once only, they will be left with their initial, partial understanding of the term or phrase. To develop deep understanding of the terms must be engaged in review activities. Offer students activities that add to their knowledge base about the terms in their notebooks. For example, they might make comparisons between selected terms; they might create analogies or metaphors for selected terms; they might simply compare their entries with those of other students. Finally, they might be engaged in games that use the terms from their academic vocabulary notebooks. After each of these activities students should be asked to make corrections, additions, and changes to the entries in their notebooks. In this way, students' knowledge of the academic terms and phrases deepen and become a sound foundation on which to build the academic content presented in class.

Algebra II

Absolute value of a complex number: (Notation: $\lvert z\rvert$) $\lvert a+bi\rvert=\sqrt{a^2+b^2}$ and is the number's distance from the origin in the complex plane.	The absolute value of $3+4i$ is: $\lvert 3+4i\rvert=\sqrt{3^2+4^2}=\sqrt{25}=5$
Amplitude: The amplitude of a periodic function is the absolute value of one-half of the difference between its maximum value and its minimum value. $A=\left\lvert\frac{\max-\min}{2}\right\rvert$	The amplitude of $y=2\sin x$ is 2. $A=\left\lvert\frac{2-(-2)}{2}\right\rvert=2$ 
Binomial Theorem: For any positive integer n: $(a+b)^n=\binom{n}{0}a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\binom{n}{3}a^{n-3}b^3+...+\binom{n}{n}b^n$ Where $\binom{n}{r}=\frac{n!}{(n-r)!r!}$ The exponents of the first term (a) of the binomial start with n and decrease by one each time. The exponents of the second term (b) of the binomial start with 0 and increase by one each time. The coefficients in the binomial expansion can also be found in Pascal's triangle.	$(x+y)^5=\binom{5}{0}x^5y^0+\binom{5}{1}x^4y^1+$ $\binom{5}{2}x^3y^2+\binom{5}{3}x^2y^3+\binom{5}{4}x^1y^4+\binom{5}{5}x^0y^5$ $=x^5+5x^4y^1+10x^3y^2+10x^2y^3+5x^1y^4+y^5$
Completing the Square: Rewriting a quadratic polynomial (by adding/subtracting constants) so that it contains a perfect square trinomial.	Solve by completing the square: $x^2+6x-5=0$ 1. $x^2+6x=5$ 2. $(x^2+6x+\underline{9})=5+\underline{9}$ 3. $(x+3)^2=14$ 4. $x+3=\pm\sqrt{14}$ 5. $x=-3\pm\sqrt{14}$
Complex Conjugates: A pair of complex numbers of the form $a+bi$ and $a-bi$ whose product will always be a real number.	$(a+bi)(a-bi)=a^2+b^2$ $(3-2i)(3+2i)=9-4i^2=9+4=13$
Complex numbers: Numbers that can be expressed in the form $a+bi$ where $i=\sqrt{-1}$ and a and b are real numbers.	Examples: $\sqrt{-4}=2i$, $6+3i$, $-1=-1+0i$

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<td>Conic Sections: The figures formed by the intersections of a plane with an infinite double cone at different angles. The conic sections are the parabola, hyperbola, ellipse and circle.</td>
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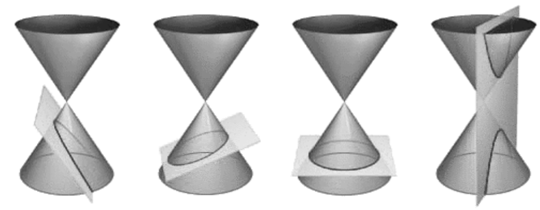

Parabola,   ellipse,   circle,   hyperbola</td>
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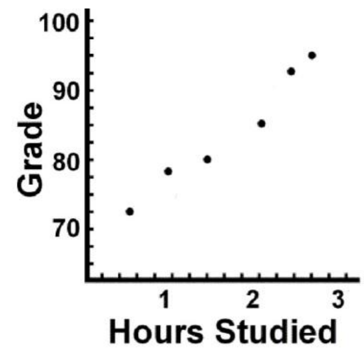
<td>Correlation: The degree to which two or more attributes or measurements on the same group of elements show a tendency to vary together.</td>
<td>Because two variables are highly correlated does not mean that one causes the other.

This graph shows a high correlation between hours studied and grades.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<td>Cramer's Rule: A method of solving a system of equations using determinants.</td>
<td>Solve for x using Cramer's rule: $\begin{array}{l} 3x+2y=7 \\ \underline{5x-2y=17} \end{array}$

$$D=\begin{vmatrix}3 & 2\\ 5 & -2\end{vmatrix}=-6-10=-16 \qquad x=\frac{D_x}{D}=\frac{-48}{-16}=3$$

$$D_x=\begin{vmatrix}7 & 2\\ 17 & -2\end{vmatrix}=-14-34=-48 \qquad y=\frac{D_y}{D}=\frac{-48}{24}=-2$$

$$D_y=\begin{vmatrix}3 & 7\\ 5 & 17\end{vmatrix}=51-35=24$$</td>
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<td>Delta Δ: A Greek letter representing an incremental change.</td>
<td>Slope is defined as the change in y divided by the change in x: $m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$</td>
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<td>Dependent/ Independent Events: Dependent events are such that the occurrence of one event affects the probability of the occurrence of the other. Two events are independent if the outcome of one does not affect the outcome of the other.</td>
<td>Tossing a coin at the same time as rolling a number cube are independent events. The result of tossing the coin has no effect on the outcome of rolling the number cube.

Taking a sock from a drawer containing several socks and not replacing it, and then taking out a second sock are dependent events. Once the first sock is removed, the sample space has changed.</td>
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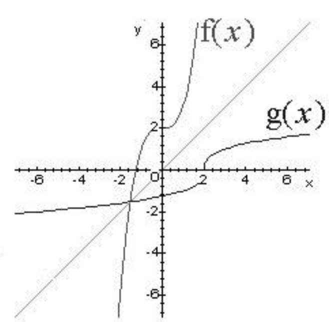
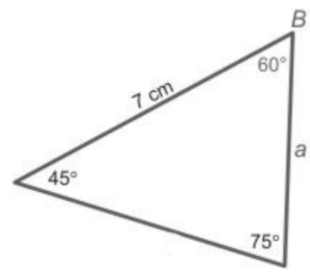
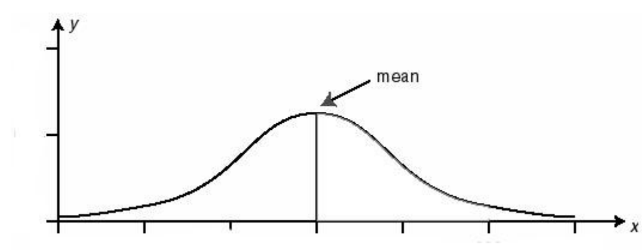
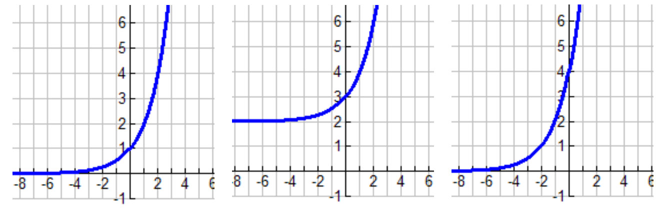
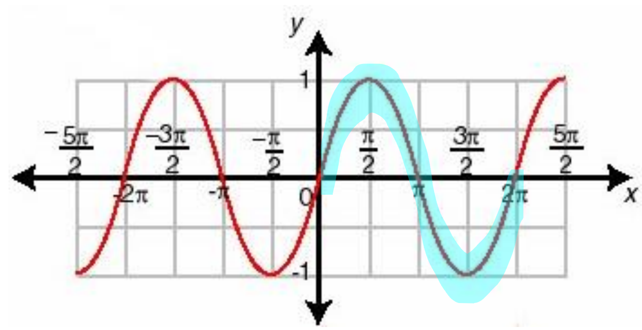
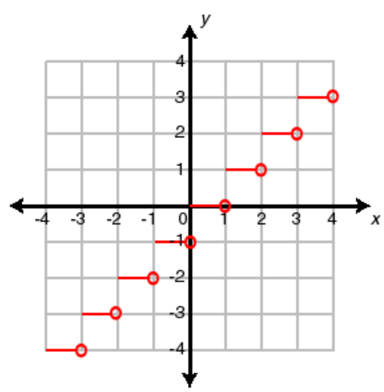
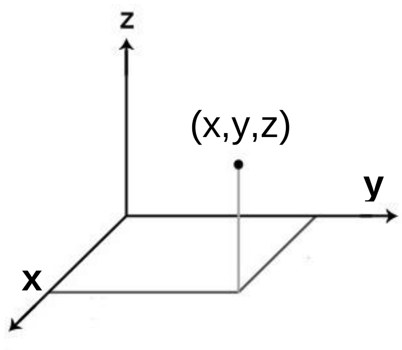
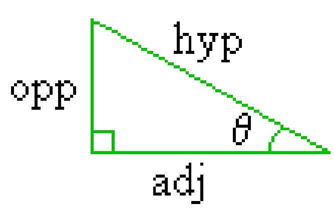
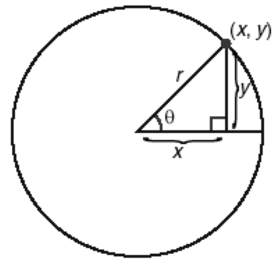
Discriminant: The discriminant is the part of the quadratic formula that in under the radical and thus indicates whether the roots of a quadratic equation are real or imaginary. $D = b^2 - 4ac$	What is the nature of the roots of the following quadratic equations? 1. $x^2 - 2x - 3 = 0$, $D = b^2 - 4ac = 4 - (-12) = 16$ D is a positive perfect square, so 2 real rational roots. 2. $x^2 - 3x - 3 = 0$, $D = b^2 - 4ac = 9 - (-12) = 21$ D is positive, so 2 real irrational roots. 3. $x^2 - 3x + 3 = 0$, $D = b^2 - 4ac = 9 - (12) = -3$ D is negative, so two complex conjugate roots. 4. $x^2 - 6x + 9 = 0$, $D = b^2 - 4ac = 36 - (36) = 0$ D is zero, so 1 real double root.
Factor Theorem: Given a polynomial P(x), then x – r is factor of P(x) if and only if P(r) = 0.	If $P(x) = (x - r)q(x)$ then $P(r) = 0$ and If $P(r) = 0$ then $P(x) = (x - r)q(x)$ For example: $x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$ so (x+1) is a factor of $x^5 + 1$ and $P(-1) = (-1)^5 + 1 = 0$
Factorial: The product of a given positive integer multiplied by all lesser positive integers: The symbol for factorial is **!**.	Four factorial: (4!) = 4 · 3 · 2 · 1 = 24.
Function Domain: The set of possible x values (the independent variable) for which the function is defined. Range: The set of all y values (the dependent variable) which are generated by the values of the domain.	If $f(x) = \|x\|$, the domain is all real numbers, the range is non-negative real numbers. If $f(x) = \frac{1}{x-2}$, the domain is all real numbers except 2, and the range is all real numbers except 0.
Functions (polynomial, exponential, logarithmic): A function is a mapping of each element of the domain to one and only one element in the range. Polynomial function of positive whole number degree n with rational coefficients: $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + ... + a_n$ Exponential function: $P(x) = ab^x$ Logarithmic function: $P(x) = \log_b x$	Polynomial function: $P(x) = x^4 - 3x^2 + 2x - 1$ Exponential function: $P(x) = 3^x$ Logarithmic function: $P(x) = \log_3 x$ (The exponential function and logarithmic function are inverses of each other)
Inverse Function: If *f* is a one-to-one function, then the inverse of *f* (f^{-1}) is the set of all ordered pairs of the form (*y*, *x*) where (*x*, *y*) belongs to *f*. The domain of *f* becomes the range of f^{-1} and the range of *f* becomes the domain of f^{-1}.	$f(x) = x^3 + 2$ and $g(x) = \sqrt[3]{x-2}$ are inverses. Their graphs are symmetric with respect to the line y= x. 

Inverse Matrix (to solve a system of equations): The inverse of matrix *A* denoted by A^{-1}, is the matrix such that $AA^{-1} = I$ (the identity matrix). The system of equations represented by $AX = B$ has the solution $X = A^{-1}B$ if *A* -is the coefficient matrix of a square system (same number of equations as variables) and -is invertible (has an inverse).	Solve $\begin{array}{l} 3x+2y=7 \\ \underline{5x-2y=17} \end{array}$ using inverse matrices. $X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 17 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$
Law of Sines: A proportion used to solve an oblique triangle. The proportion uses the ratios of the sine of an angle to the length of the side opposite the angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.	Solve for *a*: $\frac{\sin 75°}{7} = \frac{\sin 45°}{a}$, *a* = 9.56 
Logarithm: A logarithm is an exponent; $\log_b a = c$ means $b^c = a$. A logarithm written with no base is a base 10 logarithm. A logarithm written as $\ln(x) = c$ (called a natural logarithm) is a base *e* logarithm and means $e^c = x$.	$\log_2 8 = 3$ because $2^3 = 8$ $\log 100 = 2$ because $10^2 = 100$ $\ln(e^3) = 3$ because $e^3 = (e^3)$
Matrices: A rectangular array of numbers, consisting of horizontal rows and vertical columns.	The coefficient matrix of $\begin{array}{l} 3x+2y=7 \\ \underline{5x-2y=17} \end{array}$ is $\begin{bmatrix} 3 & 2 \\ 5 & -2 \end{bmatrix}$
Mutually Exclusive Events: Two events that cannot both happen at the same time. If A and B are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.	Example of mutually exclusive: A pair of dice is rolled. The events of rolling a 5 and of rolling a double have NO outcomes in common. Example of *not* mutually exclusive: A pair of dice is rolled. The events of rolling a 4 and of rolling a double have the outcome (2,2) in common.
Normal Distribution Curve: The theoretical curve where the data is distributed symmetrically about the mean. The curve is bell shaped. In a normal distribution, the mean, median and mode all coincide.	

Parent Function: The original function before any transformation is applied.	 Parent function $y = 2^x$ Vertical Translation $y = 2^x + 2$ Horizontal Translation $y = 2^{(x+2)}$
Pascal's Triangle: A triangular array of numbers where each row is generated by taking the sum of the two entries above it in the previous row. It is useful in expanding binomials and in probability.	Pascal's Triangle 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1
Period: A function has positive period b, if f(x) = f(x+b) for all x. The period of the function is the length of one complete cycle of the function.	The following function repeats every 2π units, so its period is 2π . 
Piece-wise Functions: A function defined by two or more equations over a given domain.	The absolute value function can be defined as a piece-wise function: $f(x) = \|x\| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$
Probability (theoretical, experimental): Theoretical probability is a ratio that compares the number of ways a certain event can occur to the total number of possible outcomes. Experimental probability of an event is the ratio of the number of times the event actually occurs to the total number of trials.	The theoretical probability of heads when flipping a coin is 1:2 or ½ . The experimental probability is often different in actual trials. But, the higher the number of flips the closer you come to the theoretical probability.
Pythagorean Identities: The Fundamental Trig Identity : $\sin^2 x + \cos^2 x = 1$ and the two trig identities that are derived from it: $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$.	To prove the two Pythagorean identities from the fundamental trig identity, 1) divide both sides by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ 2) divide both sides by $\cos^2 x$ to get $1 + \tan^2 x = \sec^2 x$.

Radian: A unit for measuring angles. On a circle, it is the measure of the central angle that intercepts an arc equal to the radius of the circle. There are exactly 2π radians in a circle. A radian is equal to about 57.3 degrees.	Common radian measures are $\frac{\pi}{6} = 30°,\ \frac{\pi}{4} = 45°,\ \frac{\pi}{3} = 60°,$ $\frac{\pi}{2} = 90°,\ \pi = 90°,\ \frac{3\pi}{2} = 270°,\ 2\pi = 180°$
Radical Equation: An equation that includes one or more radical expressions containing variables.	To solve a radical equation, isolate the radical and raise both sides to the power which will eliminate the radical. Be careful, because squaring, both sides of an equation can introduce extraneous solutions. So, you must check all solutions and reject those that do not solve the equation when you raise both sides to an even power.
Reciprocal Trig Identities: The reciprocal of sin(x) is csc(x), the reciprocal of cos(x) is sec(x), and the reciprocal of tan(x) is cot(x).	Example: Find x if csc(x) = 2 . If csc(x) = 2 then sin(x) = ½ . Therefore x would be $\frac{\pi}{6} = 30°$
Regression Equation: An equation that best fits data that appears to model a given curve. The regression equation attempts to find models for sets of data.	Most graphing calculators have built-in regression programs for linear, quadratic, cubic, quartic, logarithmic and exponential models.
Sampling: The process used to select a representative sample from a larger population.	Example: To try to predict the results of an election, a random sample of voters is taken. The larger the sample and the more representative the sample is of the entire population, the more accurate the prediction.
Sigma (Σ): $\sum_{k=1}^{n} p(k) = p(1) + p(2) + p(3) + p(4) + ... p(n)$	$\sum_{k=1}^{3}\left(1+\frac{1}{k}\right) = \left(1+\frac{1}{1}\right)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right) = 4\frac{5}{6}$
Step Functions: A function whose domain is made up of discrete intervals and whose value is constant over these intervals. The graph resembles a set of steps.	The greatest integer function is a step function: $[\![x]\!]$ = the greatest integer less than or equal to x 

Synthetic Division: A short-cut alternative algorithm to long divide a polynomial by a binomial.	Use synthetic division to divide $3x^3+8x^2+5x-7$ by (x+2) $$\begin{array}{r\|rrrr} -2 & 3 & 8 & 5 & -7 \\ & & -6 & -4 & -2 \\ \hline & 3 & 2 & 1 & -9 \end{array}$$ Answer: $3x^2+2x+1-\dfrac{9}{x+2}$
Three-dimensional Coordinate: Instead of an ordered pair, (x,y) as in two-dimensions, there is an ordered triple (x,y,z).	
Transcendental Function: A function that is not a polynomial function. Functions which cannot be given by any algebraic expression involving only variables and constants.	Examples of transcendental functions are: the trig functions (sin(x), cos(x), tan(x) and their reciprocals), logarithmic functions, and exponential functions (b^x).
Transformation (algebraic): The algebraic transformations include: vertical shifts, horizontal shifts, combinations of vertical and horizontal shifts, and reflections about the x-axis or the y-axis.	Given the graph of f(x): f(x) + c shifts c units up if c>0 and down if c<0 f(x+c) shifts c units left if c>0 and right if c<0 f(-x) reflects the graph over the y-axis -f(x) reflects the graph over the x-axis
Trigonometric Functions: The six functions (sine, cosine, tangent, cotangent, secant and cosecant) . They are defined in terms of ratios of sides of a right triangle. They are also called circular functions when they are defined in terms of ordered pairs on a circle on a coordinate system.	 $\sin\theta=\dfrac{opp}{hyp}$ $\cos\theta=\dfrac{adj}{hyp}$ $\tan\theta=\dfrac{opp}{adj}$  $\sin\theta=\dfrac{y}{r}$ $\cos\theta=\dfrac{x}{r}$ $\tan\theta=\dfrac{y}{x}$

Unit Circle: A circle used in trigonometry, on the Cartesian plane, with center (0,0) and radius 1. If an angle θ in standard position passes through the point (x,y) on the unit circle then $\cos\theta = x$ and $\sin\theta = y$.	Quadrant I of the unit circle with common angles in radians and their cosine(x) and sine(y) values.